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Electromagnetic

Lecture (11)

Chapter (7)

Part (2)

"The Steady State Magnetic Field"

2nd year

communications

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Lecture (11)

Part (2)

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Chapter (7)

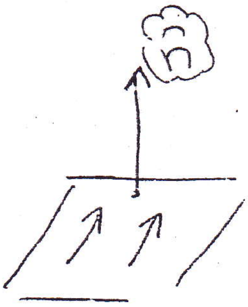
The Steady State Mag. Field

[1] The Mag. Field of a Special Case:

"An infinite surface of a Current density $\bar{K}$ = Const."

$$\boxed{\bar{H} = \frac{\bar{K} \times \hat{n}}{2}}$$

حفظ

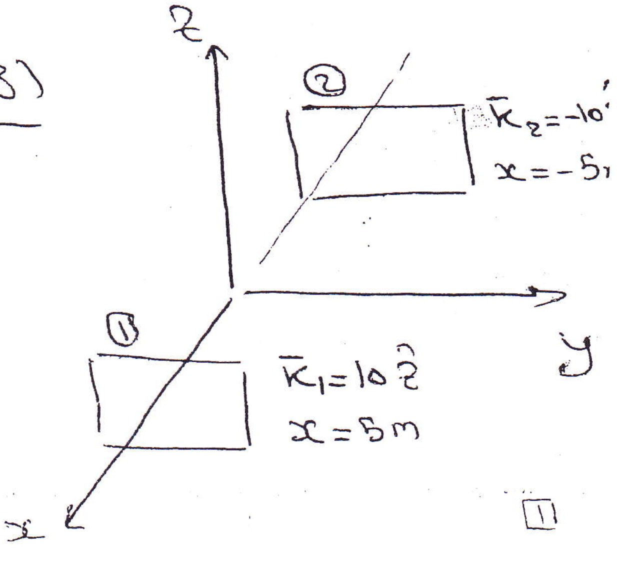


$\hat{n}$: هو متجه الوحدة العمودي على المستوى
في اتجاه النقطة التي نحسب عندها المجال.

Sheet (8)

[8]

Find $\bar{H}$ at all points ?



$$\bar{H} = \frac{\bar{K} \times \hat{n}}{2}$$

⇒ **Betn surfaces:** $-5 < x < 5$

$\hat{n}_1 = -\hat{x}$ $\hat{n}_2 = \hat{x}$

$$\therefore \bar{H} = \bar{H}_1 + \bar{H}_2$$

$$= \frac{10\hat{z} \times -\hat{x}}{2} + \frac{-10\hat{z} \times \hat{x}}{2}$$

$$= -5\hat{y} - 5\hat{y} = -10\hat{y} \ (A/m)$$

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⇒ **$x < -5$:**

$$\therefore \hat{n}_1 = \hat{n}_2 = -\hat{x}$$

$$\therefore \bar{H} = \frac{10\hat{z} \times -\hat{x}}{2} + \frac{-10\hat{z} \times -\hat{x}}{2} = 0$$

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⇒ **$x > 5$:**

$\hat{n}_1 = \hat{n}_2 = \hat{x}$ $\therefore \bar{H} = 0$

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$$\therefore \bar{H} = \begin{cases} 0 \longrightarrow & x < -5 \\ -10\hat{y} \longrightarrow & -5 < x < 5 \\ 0 \longrightarrow & x > 5 \end{cases}$$

[2] Magnetic Flux Density $\bar{B}$: (T) (Wb/m^2)

"هو عدد خطوط الفيض المغناطيسي التي تمر عمودياً على وحدة المساحات"

$$\boxed{\Phi = \int_S \bar{B} \cdot d\bar{S}}$$

$\Phi \equiv$ Magnetic flux — الفيض المغناطيسي ويقاس بوحدة ال (Wb)

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$\Rightarrow \bar{B} \propto \bar{H}$

$\therefore$ $$\boxed{\bar{B} = \mu \bar{H}}$$

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$$\mu = \mu_o \, \mu_r$$

μ → permeability

μ_o → free spale pemeability $= 4\pi \times 10^{-7}$ H/m

μ_r → relative permeability of med.

$\Rightarrow \oint_S \bar{B} \cdot d\bar{S} = 0$ مهم

" يعنى أن الفيض الكلى الخارج من أى سطح مغلقة

= الصفر "

و ذلك لأن خطوط المجال المغناطيسى دائماً هى

خطوط مغلقة:

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Sheet (9)

$\boxed{1}$ $\bar{B} = \frac{2}{\rho}\hat{\phi}$ (T)

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Find $\Phi \equiv$ flux for $0.5 \leq \rho \leq 2.5$

$0 \leq z \leq 2$

$\boxed{\text{Solution}}$

$$\therefore \Phi = \int_S \bar{B} \cdot d\bar{S} = \int_0^2 \int_{0.5}^{2.5} \frac{2}{\rho} \, d\rho \, dz$$

$$= 2 \ln \rho \Big|_{0.5}^{2.5} \cdot z \Big|_0^2$$

$$= 6.44 \text{ Wb}.$$

3| Maxwell equations:

For the steady state electric and magnetic fields:

⇛ Integral Forms:

(1) Gauss law (electrostatics):

$$\oint \bar{D} \cdot d\bar{S} = Q$$

(2) Gauss Law (Magnetostatics):

$$\oint \bar{B} \cdot d\bar{S} = 0$$

(3) Faraday Law:

$$\oint \bar{E} \cdot d\bar{L} = 0$$

(4) Ampere's Law:

$$\oint \bar{H} \cdot d\bar{L} = I$$

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⇛ point Forms :

يمكن إشتقاق ال point forms من ال Integral forms

باستخدام

Divergence theorem ←

Stokes' theorem ←

(1), (2) using Div. Theorem :

(1) $$\oint \bar{D}\cdot d\bar{S} = \int_v \nabla\cdot\bar{D}\, dv$$

$$Q = \int \rho_v\, dv$$

$$\therefore \boxed{\nabla\cdot\bar{D} = \rho_v}$$

(2) $$\oint \bar{B}\cdot d\bar{S} = \int_v \nabla\cdot\bar{B}\, dv = 0$$

$$\therefore \boxed{\nabla\cdot\bar{B} = 0}$$

(3), (4) using Stokes' Theorem :

(3) $$\oint_L \bar{H}\cdot d\bar{L} = \int_S \nabla\times\bar{H}\, dS$$

$$I = \int \bar{J}\cdot d\bar{S}$$

$$\therefore \boxed{\nabla\times\bar{H} = \bar{J}}$$

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$$(4)\quad \oint_L \bar{E}\cdot d\bar{l} = \int_S \nabla\times\bar{E}\, dS = 0$$

$$\therefore \boxed{\nabla\times\bar{E}=0}$$

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$\therefore$ Point forms:

$$\left.\begin{array}{ll}(1) & \nabla\cdot\bar{D}=\rho_v\\ (2) & \nabla\cdot\bar{B}=0\\ (3) & \nabla\times\bar{E}=0\\ (4) & \nabla\times\bar{H}=\bar{J}\end{array}\right\}\text{steady state fields}$$

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Sheet (8)

$\boxed{7}$ $\bar{H} = 4.77 * 10^4 \left[\frac{\rho}{2} - \frac{\rho^2}{0.03} \right] \hat{\phi}$

$$\bar{J} = \nabla \times \bar{H} = \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho H_\phi & 0 \end{vmatrix}$$

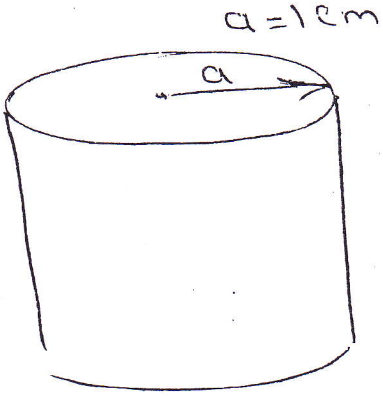
$$= \frac{1}{\rho} \left[\underbrace{- \frac{\partial}{\partial z} (\rho H_\phi) \hat{\rho}}_{0} + \frac{\partial}{\partial \rho} (\rho H_\phi) \hat{z} \right]$$

$$\therefore \bar{J} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho H_\phi) \hat{z}$$

$$= \frac{1}{\cancel{\rho}} * 4.77 * 10^4 \left[\frac{2\cancel{\rho}}{2} - \frac{3\rho^{\cancel{2}}}{0.03} \right] \hat{z}$$

$$= 4.77 * 10^4 (1 - 100\rho) \hat{z}$$

$$I = \int_S \bar{J} \cdot d\bar{S}$$



$$\therefore d\bar{S} = \rho \, d\rho \, d\phi \, \hat{z}$$

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$$\therefore \ I = 4.77 * 10^4 \int_0^{2\pi} \int_0^{0.01} (1-100\rho)\, \rho \, d\rho \, d\phi$$

Using Calculator :

$\therefore \ I = 5 \ A$

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Mansoura University
Faculty of Engineering
Communication Engineering Department

Second Year Communication
Electromagnetic Fields
Sheet (8)

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[1] Find **H** at the center of a square current loop of side *L*. See Fig.1.

[2] A current filament of 0.5 A in the a_y direction is parallel to the y axis at x = 2 m, z = -2 m. Find **H** at the origin. See Fig.2.

[3] (a) Find **H** in Cartesian components at p(2, 3, 4) if there is a current filament on the z axis carrying 8 mA in the a_z direction.

(b) Repeat if the filament is located at x = -1, y = 2.

(c) Find **H** if both filaments are present.

[4] Use Ampere's law to obtain **H** due to an infinitely long, straight filament placed on z-axis of current I_o.

[5] A thin cylindrical conductor of radius a, infinite in length carries a current I_o in the a_z direction. Find **H** at all points using Ampere's law.

[6] Determine **H** for a solid cylindrical conductor of radius a. Assume uniform distribution of the current I_o in the a_z direction.

[7] A cylindrical conductor of radius 1 cm has an internal magnetic field $H = (4.77 \times 10^4)(\frac{\rho}{2} - \frac{\rho^2}{3\times10^{-2}})a_\varphi$ (A/m). What is the total current in the conductor?

[8] A current sheet, K = 10 a_z A/m, lies in the x = 5 m plane and a second sheet, K = -10 a_z A/m, is at x = -5 m. Find **H** at all points.

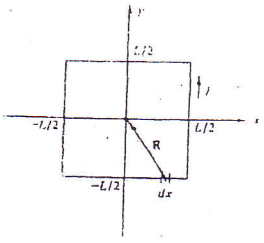

Fig.1

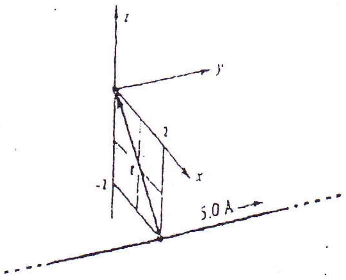

Fig.2

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